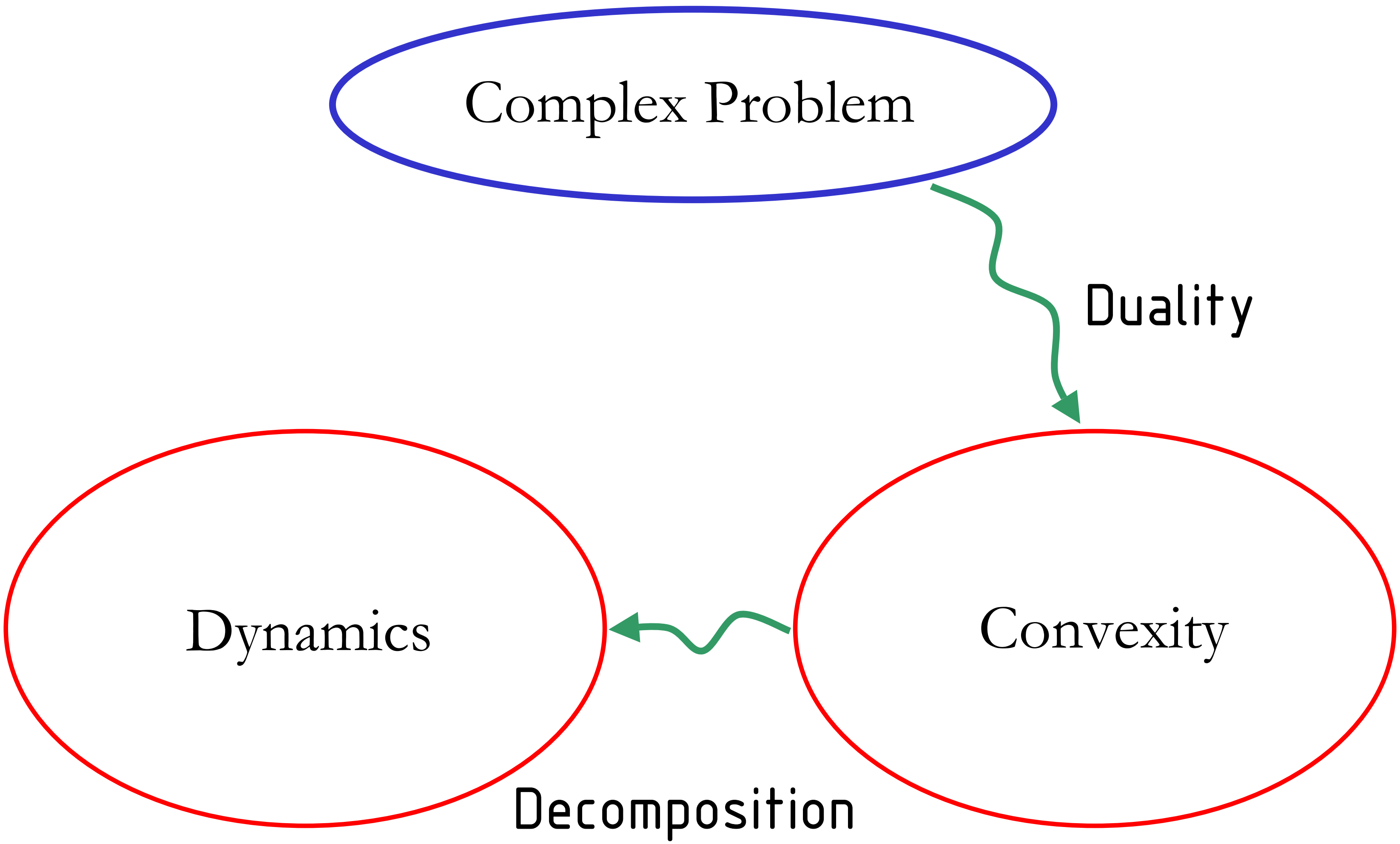
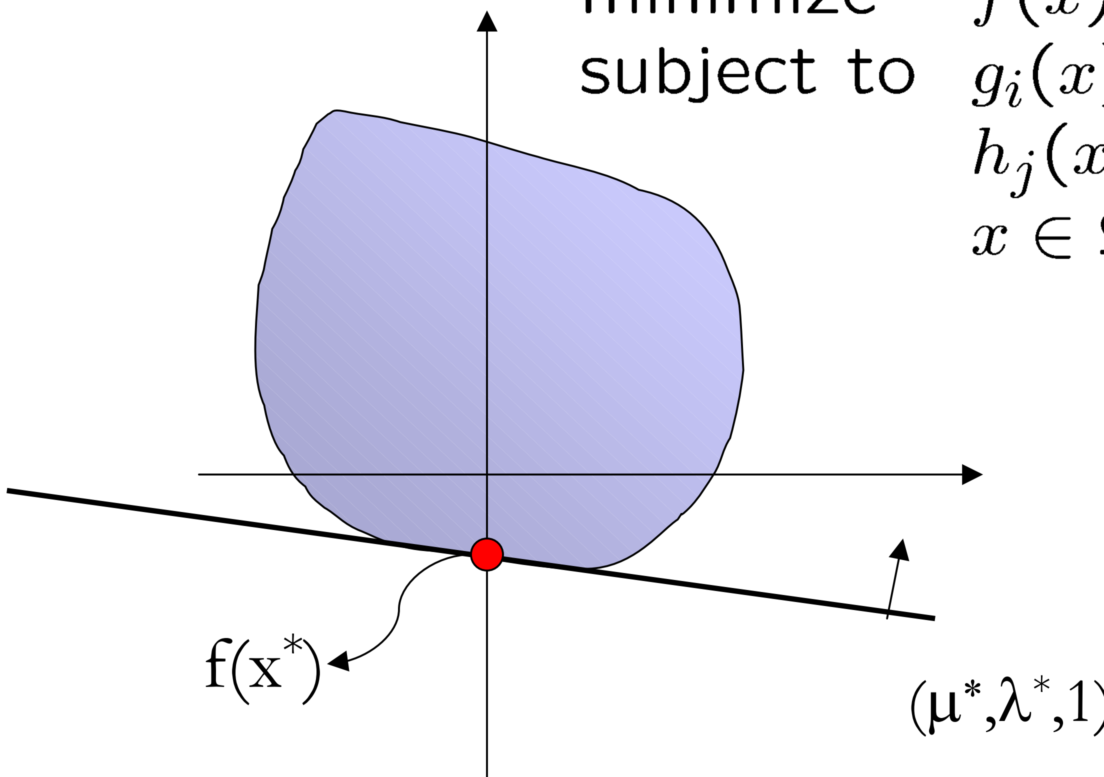

Complex Problem
Duality
Dynamics
Convexity
Decomposition

Duality

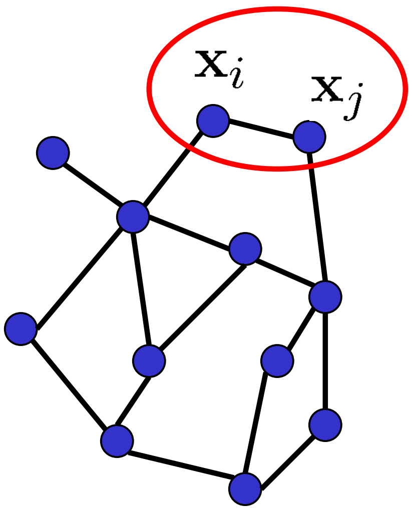
$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & g_i(x) \leq 0 \\ & h_j(x) = 0 \\ & x \in \Omega \subset \mathbb{R}^n \end{array}$$

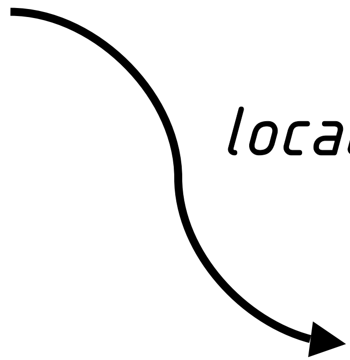


$$\inf_{x \in \Omega} \sup_{\mu > 0, \lambda} \mathcal{L}(x, \mu, \lambda) \qquad \sup_{\mu > 0, \lambda} \inf_{x \in \Omega} \mathcal{L}(x, \mu, \lambda)$$

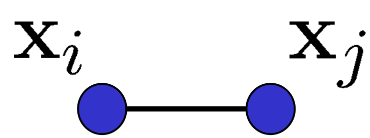
Decomposition

Dual: $\max \sum_{i,j} \mathbf{q}_{ij}(\nu)$





Idea: *Solve master problem with subproblem dynamics*



$$\mathbf{q}_{ij}(\nu) = \min f(\mathbf{x}_i, \mathbf{x}_j)$$

- Local Constraints ⇒ Distributed Algorithm
- Predictable Global Behavior

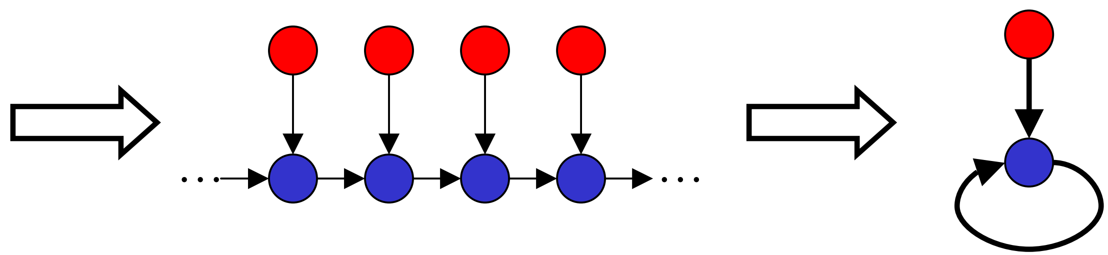
Example 1: LFSR Acquisition

$$\max_p -\mathbb{E}_p[\psi(\mathbf{x})] + \mathcal{H}_p[\mathbf{x}]$$

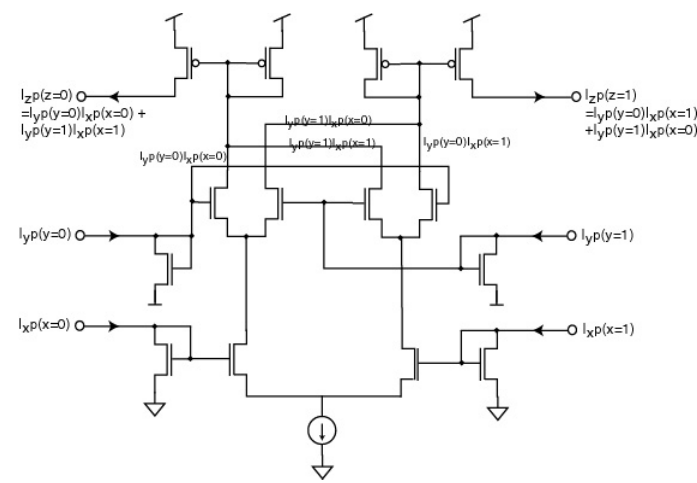
Maximize Likelihood

Duality → Belief Propagation/Extended Kalman Filter

$$\mathbf{x}_k = \mathbf{x}_{k-1} \oplus \mathbf{x}_{k-T}$$
$$\mathbf{y}_k = \mathbf{x}_k + \omega_k$$



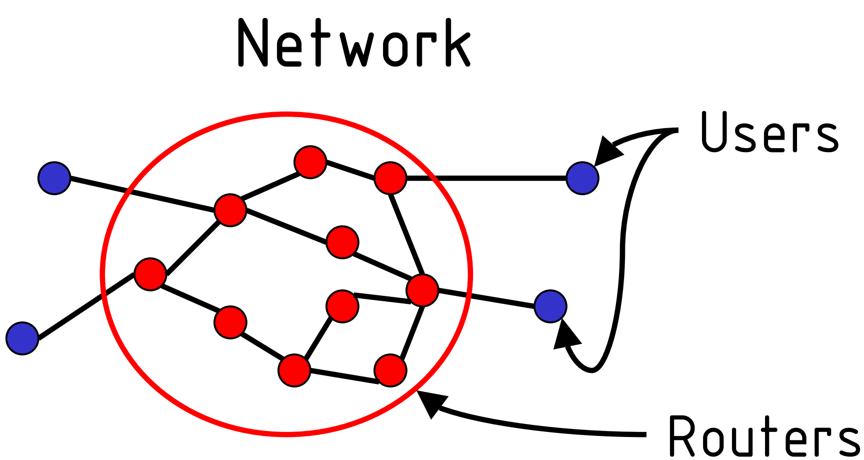
- Bounds on optimality
- Decomposition allows implementation in analog hardware



Example 2: TCP/AQM

$$\max_{\mathbf{x}} \quad \sum_i U_i(x_i)$$
$$\text{s.t.} \quad \mathbf{R}\mathbf{x} \preceq c$$

Maximize Utility subject to Capacity constraints



Users adjust to *prices*

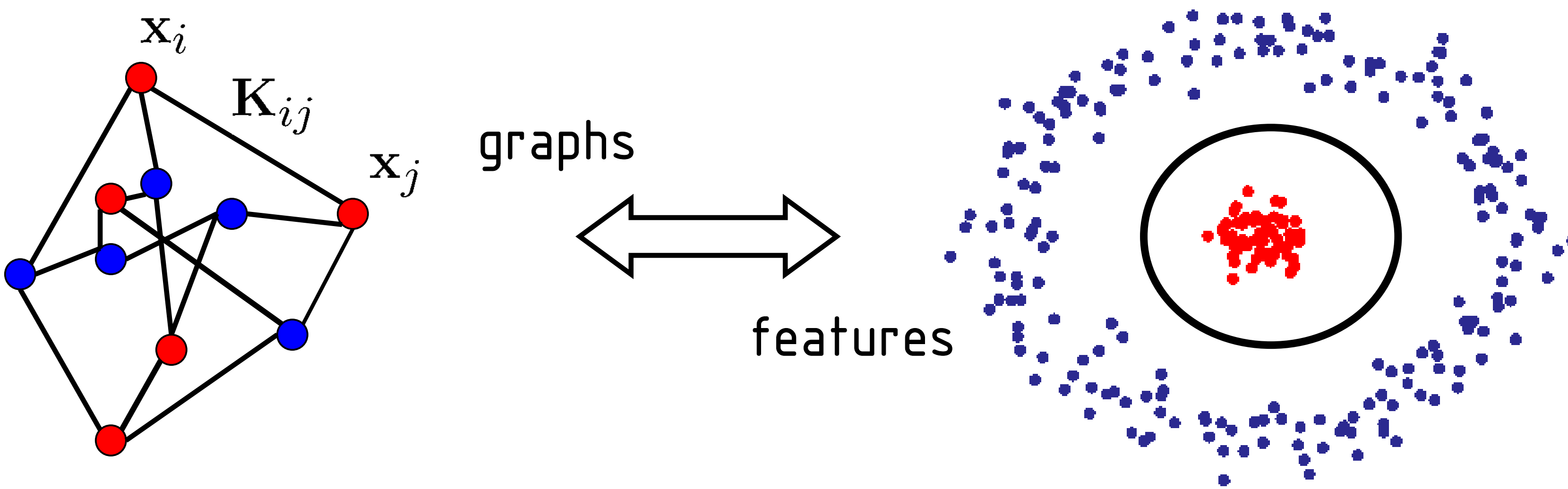
Routers adjust to *rates*

- Becomes standard congestion control protocol
- Provably stable under arbitrary delays

Example 3: Data Clustering

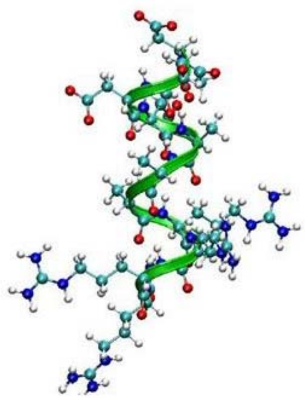
$$\min_{\mathbf{y},f} \sum_i \mathbf{V}(\mathbf{y}_i, f(\mathbf{x}_i)) + \lambda \|f\|_{\mathbf{K}}^2$$

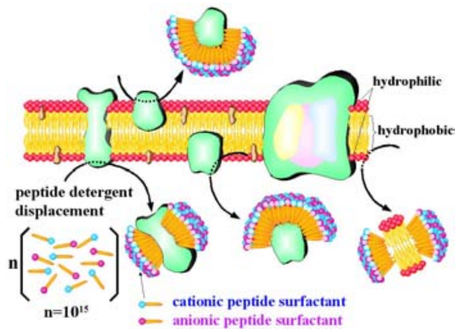
Minimize Distortion,
Maximize Smoothness

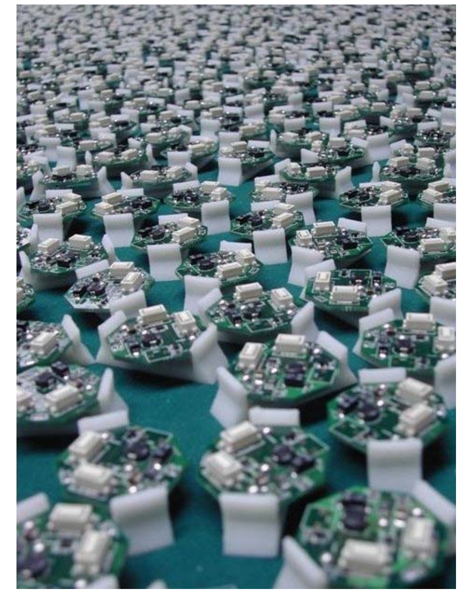


- Generally NP-HARD
- Dual Provides Convexity
- Decomposition Provides Implementation

A bevy of complex problems







- Lots of interacting parts
- Local Constraints
- Global Behavior
- Dual Decomposition?